Dr. R. Glass

MAT 225 - Take Home

No Math lab help!

1. (10 points) From the Theory of Thermodynamics the enthalpy (*H*) of a system is given by $H = U + PV$ where *U* is energy, *P* is pressure and *V* is volume. By making use of the combined First and Second Laws of Thermodynamics that $dU = TdS - PdV$ where *S* is entropy and *T* is temperature, find *dH* in terms of temperature (*T*), entropy (*S*), pressure (*P*) and volume (*V*).

2. (10 points) Compute $\int_C \vec{F} \bullet d\vec{r}$ where C is the path generated by taking a wheel of radius **b** units and placing a pen **a** units from the center (initially placed vertically beneath the center - **Figure 4**) and rolling the wheel right through 2 revolutions $\vec{F}$ is the force field equal to **(x,y)**. Assume the x axis - horizontal, y vertical.

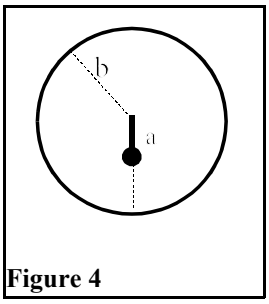

Figure 4

3. (10 points) Compute $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3}\,dxdy$ algebraically (without using a table of integrals or calculator).

4. (20 points). A boat race is to take place on a river several miles wide that flows from west to east. The current in the river is **proportional to the square of the distance** in miles from the southern bank a**t a rate of 10 mph per mile**. Prior to the start of the race, a wind develops from the southeast. The wind speed at a point (*x,y*) on the river has magnitude $x^2 + y^2$ mph.
   a. What is the resulting force field for a boat at any point on the river?
   b. What is the work done traveling clockwise circular path of radius 1 centered at (1,0) from the point (0,0) to the point (2,0)?

5. (10 points) Let the density of particles in space be given by $\rho(x,y,z) = (x^2 + y^2 + z^2)^{-1/2}$ particles per meter^3. Find the total number of particles in the region bounded by the xz plane, xy plane, the plane y = x and the surface $x^2 + y^2 + z^2 = 9$.

6. (10 points) Find $\iint_R f(r,\theta) r\,dr\,d\theta$ where R is the region in **Figure 1** and $f(x,y) = x^2y + xy^2$. Polar coordinates required for all integrals..

7. (20 points) Let $f(x,y) = xy - x^3 - y^2$
   a. Find all relative extrema.
   b. Find the absolute maximum and minimum in the region bounded by y = x and $y = x^2$.

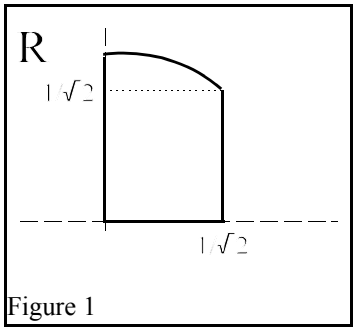

Figure 1